Intensions, extensions, and quantifiers

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Suppose we want to follow Frege and distinguish an expression's *denotation* from its *sense*. Suppose also we take the denotation of a predicate to be its extension: the set of its instances. The following argument appears to show that this leads to trouble.

1. All humans are featherless bipeds, and all featherless bipeds are human, but there could have been featherless bipeds that are not human. In short, (Ax)(Hx <-> FBx) & <> (Ex)(~Hx & FBx)).
2. By existential generalisation over the predicate positions, it follows that (EX)(EY)((Ax)(Xx <-> Yx) & <> (Ex)(~Xx & Yx)).
3. If things in predicate position denote sets of individuals, this can be read as: there is a set X and a set Y such that X and Y have the same members and it is possible for something to be a member of Y and not of X.
4. But if X and Y have the same members, then they are identical; and then nothing could belong to "one of them" without also belonging to "the other".
5. Hence things in predicate position do not denote sets of individuals.

The argument is modeled on a brief passage (p.13) in Tim Williamson's latest paper on the Barcan Formula. Williamson there argues against the plural interpretation of second-order quantifiers. On this interpretation, the sentence in (2) can be read as "there are things xx and things yy such that all xx's are yy's and all yy's are xx's and it is possible for something to be one of the yy's but not of the xx's". Williamson objects that if the xx's *just are* the yy's, then it is not possible for something to belong to "the ones" without also belonging to "the others".

I'm not particularly hostile to the conclusion of these arguments: that predicates denote intensions (properties) rather than extensions, but the argument seems a bit fishy. On the other hand, I'm not sure quite what's wrong with it.

One problem is that it seems to overgeneralise. It appears to show that all expressions (not just predicates) denote their intension. For instance, using propositional quantifiers, it would show that sentences do not denote truthvalues: (Ep)(Eq)((p <-> q) & <>(p & ~q)), but it is not true that there is a truth-value p and a truth-value q such that p = q and yet it is possible for p to be T and q to be F.

Worse, suppose we treat definite descriptions as complex singular terms. Then we can show that "the present Pope" does not denote Joseph Ratzinger: from "the Pope = Ratzinger & <>~(the Pope = Ratzinger)", we get "(Ex)(Ey)((x = y) & <>~(x=y)", which we reject on the account that if x *just is* y, then surely nothing could be identical to "one of them" without being identical to "the other".

Or consider plural terms: it seems true that the featherless bipeds are the humans while it is possible that the featherless bipeds are not the humans. By existential generalisation, we reach the allegedly absurd claim that there are things xx and things yy such the xx's are the yy's and and yet it is possible for something to be one of the xx's but not of the yy's. We would conclude that "the featherless bipeds" does not denote the featherless bipeds.

So what's wrong with the argument? There are several vulnerable spots, and perhaps different ones are relevant for different applications.

One might reject the validity of existential generalisation at step 2. It is doubtful whether "the shortest spy is a woman, but it could have been a man" entails "there is something that is a woman and could have been a man". But we could also accept this step and reject the step 4, which invokes some form of Necessity of Identity. Merely rejecting the Necessity of Identity however won't get at the heart of the problem. We would have to say that the relevant bound variables are not directly referential: maybe they come with an associated counterpart relation, or an associated intension. I guess that would have been the response of Carnap and Church. (Interestingly, Church discusses something like this argument in his 1943 review of Quine's "Notes on Existence and Necessity", and suggests that the argument goes through in the first-order case, but not in the second-order case.) Alternatively, one might try to reject the middle step 3, which assumes that whenever expressions of type X denote entities of type Y, then one can a quantifier into the position of such an expression as "there is a Y". This won't work for the plurals and descriptions however.